### Musculoskeletal Model Dynamics

In contrast to inverse dynamics where the motion of the model was known and we wanted to determine the forces and torques that generated the motion, in forward dynamics, a mathematical model describes how coordinates and their velocities change due to applied forces and torques (moments).

From Newton’s second law, we can describe the accelerations (rate of change of velocities) of the coordinates in terms of the inertia and forces applied on the skeleton as a set of rigid-bodies:



Multibody dynamics

where  is the coordinate accelerations due to joint torques, , Coriolis and centrifugal forces, , as a function of coordinates, , and their velocities, , gravity, , and other forces applied to the model, , and  is the inverse of the mass matrix.



Moments due to muscle forces



Muscle contraction dynamics

Muscle activation dynamics



The net muscle moments, , in turn, are a result of the moment arms, , multiplied by muscle forces, , which are a function of muscle activations, , and muscle fiber lengths, , and velocities, . Muscle fiber velocities are governed by muscle contraction dynamics, , which is dependent on the current muscle activations and fiber lengths as well as the coordinates and their velocities. Activation dynamics, , describes how the activation rates, , of the muscles respond to input neural excitations, , generally termed the model’s controls. These form a set of differential equations that model *musculoskeletal dynamics*.

### States of a Musculoskeletal Model

The *state* of a model is the collection of all model variables defined at a given instant in time that are governed by dynamics. The model dynamics describe how the model will advance from a given state to another through time. In a *musculoskeletal model* the states are the coordinates and their velocities and muscle activations and muscle fiber lengths. The dynamics of a model require the state to be known in order to calculate the rate of change of the model states (joint accelerations, activation rates, and fiber velocities) in response to forces and controls.

### Controlling a Musculoskeletal Model

The forces (e.g., muscles) in a *musculoskeletal model* are governed by dynamics and have inputs that affect their behavior. In OpenSim, these inputs are called the *controls* of a model, which can be excitations for muscles or torque generators. Ultimately, controls determine the forces and/or torques applied to the model and therefore determine the resultant motion.

### Numerical Integration of Dynamical Equations

A simulation is the integration of the musculoskeletal model’s dynamical equations starting from a user-specified initial state. After applying the controls, the activation rates, muscle fiber velocities, and coordinate accelerations are computed. Then, new states at small time interval in the future are determined by numerical integration. A 5th-order Runge-Kutta-Feldberg integrator is used to solve (numerically integrate) the dynamical equations for the trajectories of the musculoskeletal model states over a definite interval in time. The Forward Dynamics Tool is an open-loop system that applies muscle/actuator controls with no feedback, or correction mechanism, therefore the states are not required to follow a desired trajectory.